AN EXAMINATION OF THE PARKING SITUATION AT BROWN UNIVERSITY

HITCHHIKER’S GUIDE TO PARKING AT BROWN

January 16, 2021

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Dear President Paxson and Director Lighty,

We are a group of Brown University students participating in the Brown Mathematical Contest in Modeling, and we have built a mathematical model to analyze the current parking situation at Brown. We are writing to share our findings of the Brown parking space allocation and suggest improvements.

It's no news that faculty, staff, and students at Brown all complain about the difficulty of getting a parking space on campus. We have investigated this claim quantitatively. First, we divided the Brown Campus into 5 "districts": 3 in central campus and the other 2 covers the Warren Alpert Medical School area and the area of Nelson Fitness Center and beyond. Using real data from Brown's websites, we analyzed the relationship between the number of existing parking spots and the demand for parking spaces. We have found that supply is greater than demand in all 5 districts.

Therefore, it is essential that we wisely allocate the existing parking slots to benefit the brown community and visitors. To solve this challenging problem, we built a mathematical model aiming to create an allocation strategy that not only maximizes the number of occupied parking slots but also generates maximum profit for the Brown Transportation Office. In this model, we consider the needs of a diverse group of people: students, faculty, and visitors, as well as how seasons and particular days of the week affect their needs for parking spaces.

For individual parking space permits, we find it optimal to first fill the need of Brown faculty and staff, and then issue the rest to students in need if there is still space. For visitor parking at Brown, we find it optimal to set aside a different number of visitor parking slots based on the season and the day of the week (around 20 in the fall and 200 in the spring). The exact parking spot allocation strategy for each of the 5 districts can be found in Section 4 of this report.

While our strategy does maximize the number of parking spots occupied and Brown's profit, we acknowledge that some important factors, such as the fairness of assignment among different classes of people, were left out when creating this strategy. Therefore we outlined several possible improvements to our strategy in Section 5, based on those factors.

Furthermore, since our finding indicated a shortage of current parking spaces, we investigated the option of building new parking lots on campus. We weighed the cost of building new parking lots against the potential income generated from those added parking spaces in a year. We concluded that Brown should build as many new parking slots as possible on the districts north of Waterman Street, and none in the area south of Waterman street. This will maximize Brown's parking income while providing more parking spaces to meet the demand.

We hope the results we have shared above would be of value to the university and of benefit to the whole of Brown community.

Sincerely,
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1 Introduction

Parking is a difficult problem in many population dense areas [6]. At Brown University, there has always been complaints from student, faculty, and staff about the shortage of parking spaces.

Brown University spans 146 acres of land [9] in Providence, Rhode Island. With a small campus hosting more than 10000 students and 4000 employees, Brown has very limited parking spots. Brown's unique campus location also brings significant challenges for parking. The Brown stadiums are at far North, and the Warren Alpert medical school is located the city of Providence in the South. This stretch of campus creates demand for parking at different locations of campus. For example, undergraduate students might want to park around central campus near their dorms, whereas medical students would want to park in the city near the medical school. It is essential that everyone's different needs are taken into account when designing the solution to the parking problem.

Space is not the only factor that needs attention. As a member of the Ivy League, Brown attracts many visitors from afar with its prestige and rigor. The problem of visitor parking also needs to be carefully examined. We note that different number of visitors come in different seasons of year and time of day, both of which present unique challenges for visitor parking.

In this project, we examine the supply and demand of parking spaces at Brown across physical locations and time, and develop a strategy for allocating parking space to different classes of people. The goal of this strategy is to maximize the number of occupied parking spots as well as the profit generated, with special attention to the needs of different classes of people and their demand at different time. Moreover, we also propose a strategy for where and how much new parking spots should be built.

There has been some existing work on using mathematical approaches to optimize parking space utilization. Shao et al. [8] proposed a reservation and allocation model to share private parking spaces and achieve maximum usage of resources. However, their research is limited to urban-commuter-owned parking spaces and is not applicable to Brown University's special situation. Geng et al. [1] also designed a smart parking system using Mixed Integer Linear Program. Our work is different in that we consider the existence of yearly-permit parking in a campus setting.

2 Assumptions

In this section, we present the assumptions we make for our model and the scope of our project. This section is organized as follows, in Section 2.1, we present the definitions of variables in our model and their scope. Section 2.2 and 2.3 present the supply of existing parking slots at Brown and the demand for spots from drivers. Section 2.4 presents the parking rates we use in the model as well as the capital needed to construct new parking lots.

2.1 Definition & Scope

2.1.1 classes of people

First, we define the 7 different classes of people we are considering in our model for the allocation of parking spots. They are: 1) undergraduate students, 2) graduate students, 3) medical students, 4) academic faculty, 5) emeriti, 6) staff, and 7) visitors at Brown. We further limit visitors to only campus tourists. We make this assumption because this is the only group of visitors that vary greatly at different time. Other academic visitors are usually a constant factor all year round, and we incorporate them into the academic faculty class. For simplicity, we assume that all classes except for visitors are evenly distributed across campus.

2.1.2 model of campus

Then, we define a simplified model of the Brown University campus. For simplicity, we separate campus into 5 main “districts”, and consider the supply and demand of parking in each of those districts and the relationship between districts. The 5 districts are defined in table 1:

District	Details
Nelson	the area north of the Nelson Fitness center on Cushing St.
Central-North	the area south of Cushing street, north of Waterman St.
Central-Mid	the area south of Waterman St., north of George St.
Central-South	the area south of George St., north of Williams St.
Downtown	the area south of Williams St.

Table 1: A five-district model of Brown University Campus

2.1.3 timing

Finally, we define the time of the year we consider in our model. We limit the scope of our model to only consider Spring and Fall when the majority of the Brown community are on campus. This assumption is reasonable because during winter and summer break, finding a parking spot will no longer be a difficult task when the demand is low. We assume that there are 14 and 15 weeks in a typical fall and spring semester at Brown, respectively.

2.2 Supply

According to the parking map B provided by the Brown Department of Facilities Management on April 2020, we estimate Brown’s current parking spaces supply in Table 2.

2.3 Demand

Before estimating the demand, we make some general assumptions to simplify the model:

Region	Number of existing parking spots
Nelson	300
Central-North	200
Central-Mid	100
Central-South	460
Downtown	600

Table 2: Supply of existing parking spots on campus. Data based on academic year 2019-2020

- All students (undergraduates, graduate students, medical students) are willing to park in the neighboring district, while all Brown employees (faculty, staff, emeriti) are only willing to park in the district of their work location.
- All visitors must park in the Central-South district, and we can allocate different number of spots for visitors according to the time of the year and time of the week. This assumption is reasonable because the current visitor parking lot Power Street Parking Garage is in the Central-South district.
- All visitors are campus tourists and thus do not park overnight. The allowed parking time is from $8am$ to $6pm$ each day.
- Street parking is not considered; only permanent parking and visitor parking is considered.

Group	Number of People
academic faculties	1508
undergraduate students	7160
freshman students [2]	1675
staff	3201
graduate students	3173
medical Students	598
medical academic faculties	599
medical clinical facilities	1737
campus tourists (fall weekday)	60
campus tourists (fall weekend)	30
campus tourists (spring weekday)	800
campus tourists (spring weekend)	400 [3]

Table 3: Population data for estimating demand, gathered from the Brown Office of Institutional Research [3] [4] and the Warren Alpert Medical School official web page [7]

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[2] this statistic, used along the freshmen return rate of 98 %, is used to estimate the number of junior and seniors

[3] the campus tourists numbers are estimated according to the statistics of the Brown Admissions Office [5]: there are

Next, we focus on the data provided by official Brown websites. Table 3 summaries relevant data based on academic year 2019-20, and we assume that:

- The average sabbatical rate for academic faculties is 12%, and those who are on sabbatical will not need parking lots.
- 1% of junior and senior undergraduate students owns a car and needs a parking slot.
- 20 emeriti demands a parking spot on campus.
- 20% of graduate students and medical students need a parking spot.
- Academic faculties, medical academic faculties, and medical clinical faculties have faculty-to-car ratio 35%, 40%, and 10%, respectively.
- 15% of the staff drive to work.
- ICERM center has 1000 visitors for each season, 10% of whom owns a car and needs parking spaces downtown.
- 50% of the visitors (campus tourists) drive to campus and have parking needs.
- 70 university-owned vehicles needs to be parked.

From these assumptions, we obtain table 4 of demand for each class of drivers.

Group	Demand for parking spaces
Undergraduate students	39
Graduate students	635
Medical students	120
Academic faculties	449
Medical academic faculties	240
Medical clinical facilities	174
ICERM academic visitors	100
Emeriti	20
Staff	481
campus tourists (fall weekday)	30
campus tourists (fall weekend)	15
campus tourists (spring weekday)	400
campus tourists (spring weekend)	200

Table 4: Demand of parking spots

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4 tour slots each weekday and 2 each Saturday and Sunday; there are about 15 visitors per slot during the fall, and 200 during the spring

2.4 Rates

First, we consider the rates charged for parking.

First, we reduce the 7 different classes of drivers to 3 groups: employees (faculty, emeriti, and staff), students (undergraduates, graduate students, and medical students), and visitors.

For simplicity, we assume drivers in the same group are charged the same price for parking, as shown in Table 5.

Group	Hourly Rate ($)
employees	0.25
students	0.19
visitors (weekdays)	3.00
visitors (weekends)	2.00

Table 5: hourly rate for parking.

Then, we make assumptions for the cost of building new parking spots.

In a real-world pavement of new parking spaces, a lot of different factors can influence the cost, for example different labor inputs and the types of materials used. To simplify the model, we only consider labor input(L) and capital input(K) as influential factors of the cost of new parking spaces. For labor input, suppose each unit of labor cost a wage rate of $\$12.00$ per hours, and it takes a unit labor 8 hours to build 1 new parking slot. For capital input, suppose the average area of each new parking lot is 200 square feet, with each square feet costing $\$2.40$ as capital.

2.5 Assumptions made on visitors

1. The rate (number of visitors/hour) in which the visitors arrive at Brown with respect to the time of the day, t, on a given day, satisfies the shape of a normal distribution with mean at 11 a.m., variance 1, disregarding the tail where $t <$ 8 a.m. and $t >$ 6 p.m.
2. The rate (number of visitors/hour) in which the visitors leave Brown with respect to the time of the day, t, on a given day, satisfies the shape of a normal distribution with mean at 1 p.m., variance 1, disregarding the tail where $t <$ 8 a.m. and $t >$ 6 p.m.

The above two assumptions are fairly reasonable since the visitors are likely to arrive early and leave late.

3 Model

3.1 Allocate Parking Spaces for all Regions but South Campus

In this section we describe the model we use to allocate parking spaces for the four regions on campus in which we only need to allocate permanent parking spaces, namely the Medical School District, North Campus, Center Campus, and Nelson Area, in a given season.

First off, we outline some data of our interest:

1. Total number of hours in the season: H
2. Number of available parking slots in the region: K
3. Total number of faculties and staffs in the region: N_1
4. Total number of student in the region: N_2
5. Hourly parking rate for faculties and staffs: r_1 (USD/hour)
6. Hourly parking rate for students: r_2 (USD/hour)

Those data can all be calculated directly from our assumption made in section 2. K can be found in table 2, N_1 and N_2 can be found in table 4, and r_1, r_2 can be found in table 5.

The variables of our concern are x_1: number of parking spaces allocated to faculties, and x_2: number of parking spaces allocated to students.

Since the demand and supply of parking spaces are fixed by the parking rates, we only need to maximize the hourly profit. Hence, our goal becomes that of maximizing the profit $\sum_{1 \leq i \leq 2} x_i r_i$ given the following constraints:

1. $\forall 1 \leq i \leq 2, x_i \leq N_i$
2. $\sum_{1 \leq i \leq 2} x_i \leq K$

Claim 3.1.1. *$\sum_{1 \leq i \leq 2} x_i r_i$ is maximized given the above constraints whenever $\sum_{1 \leq i \leq 2} x_i = \min(\sum_{1 \leq i \leq 2} N_i, K)$ and $x_1 = \min(N_1, K)$.*

Proof. Intuitively, since $r_1 \geq r_2$ always holds, prioritizing faculty parking spaces is always the optimal solution. More formally, if $\sum_{1 \leq i \leq 2} x_i < \min(\sum_{1 \leq i \leq 2} N_i, K)$, we can still satisfy the constrains by increasing x_1, thus increasing $\sum_{1 \leq i \leq 2} x_i r_i$. Similarly, if $x_1 < \min(N_1, K)$, we can still satisfy the constraints by increasing x_1 by 1 and decreasing x_2 by 1, also increasing $\sum_{1 \leq i \leq 2} x_i r_i$. On the other hand, if both $\sum_{1 \leq i \leq 2} x_i = \min(\sum_{1 \leq i \leq 2} N_i, K)$ and $x_1 = \min(N_1, K)$, we cannot increase the profit any further since both x_1 and $\sum_{1 \leq i \leq 2} x_i$ are maximized. $\square$

3.2 Allocate Parking Spaces for South Campus: Accounting for Visitor Parking

When allocating parking spaces for South Campus, besides the data and variables described in the section 3.1, i.e. $K, N_1, N_2, r_1, r_2, x_1, x_2$, we also need to take into account x_3, how many parking spaces to leave to visitors. To do this, we first need to model the visitor populations. According to the assumptions made in section 2.5, we have the rate in which visitors come to Brown as a function of the time of the day, $v_{in}(t)$, as well as the rate in which visitors leave Brown as a function of the time of the day, $v_{out}(t)$, both of which have the shape of a normal distribution: $v_{in}(t) = c_1 e^{-\frac{1}{2}(x-11)^2}$ and $v_{out}(t) = c_2 e^{-\frac{1}{2}(x-13)^2}$, where $8 \leq t \leq 18$.

To determine the constants c_1, c_2, we need to use the total number of visitors in the season, S, from table 3. Solving $\int_8^{18} v_{in}(t)\ dt = \int_8^{18} v_{out}(t)\ dt = S$ gives us the exact information of c_1 and c_2, and hence the formula for v_{in} and v_{out}.

With v_{in} and v_{out} at hand, we can then express $X(t)$, the number of visitors on campus at a given time of the day by taking the difference of the two integrals: $X(t) = \int_8^t v_{in}(t)\ dt - \int_8^t v_{out}(t)\ dt$.

Let r_3 be the hourly visitor parking rate from table 5, and x_3 be the number of parking slots reserved for the visitors. Our goal becomes finding x_1, x_2, x_3 such that the total profit earned in a day,

$$r_3 \int_8^{18} \min(X(t), x_3)\ dt + 10 \sum_{1 \le i \le 2} x_i r_i$$

is maximized under the following constraints:

1. $\forall 1 \le i \le 2, x_i \le N_i$
2. $\sum_{1 \le i \le 3} x_i \le K$

The above equation is difficult to maximize, but since parking fees are charged by hour blocks, in practice we can approximate the integral by a summation, and then enumerate all the possibilities of x_3 to find the optimal value. See Code A for detail.

3.3 Adding Parking Space Model

In this section, we consider the possibility of adding new parking slots on Brown's campus. We create the following model to compare the cost of building new parking lots and and the added income from those new parking lots.

Before introducing the model, we first define some essential terms and their relations.

Definition 3.3.1 (Production function). *Production function is a real-valued (number of products produced) function where number of different inputs as variables.*

Definition 3.3.2 (Cost function). *Cost function is a real-valued (money) function where the number of quantities as variables.*

Definition 3.3.3 (Returns to scale). *Suppose $f(x, y)$ is a production function of some product, $a > 1$ is arbitrarily chosen, the situation is called:*

- *(1)increasing returns to scale if:* $f(ax, ay) > af(x, y)$
- *(2)decreasing returns to scale if:* $f(ax, ay) < af(x, y)$
- *(3)constant returns to scale if:* $f(ax, ay) = af(x, y)$

By simple computations, we have the following theorem:

Theorem 3.3.1. *Suppose a product has a Cobb-Douglas production function, i.e.,*

$$x(L, K) = L^{\alpha} K^{\beta} \tag{3.3.1}$$

where α, $\beta > 0$, *then:*

- (a) $\alpha + \beta > 1 \Rightarrow$ *increasing returns to scale*
- (b) $\alpha + \beta = 1 \Rightarrow$ *constant returns to scale*
- (c) $\alpha + \beta < 1 \Rightarrow$ *decreasing returns to scale*

Recall: in our cases, $w_L = 12 \times 8 = 96$, $w_K = 2.4 \times 200 = 480$, where w_L and w_K are costs for single unit of labor and capital input, respectively.

We assume that the production function is Cobb-Douglas; moreover, the building of new parking lots should exhibit an increasing returns to scale slightly, hence

$$x(L, K) = L^{\alpha} K^{\beta} \tag{3.3.2}$$

where $\alpha + \beta > 1$ slightly.

To minimize the cost, we apply Lagrange multiplier method to get the following theorem:

Theorem 3.3.2. *f(x,y) is a continuously differentiable production function such that:* $\frac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}$ *is a decreasing function with variable* x, $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} > 0$, *and:* $ax + by = I$, $a, b, I \in \mathbf{R}$, *then: the solution(s) of the following system of equations maximizes production on the line* $ax + by = I$.

$$\begin{cases} \dfrac{\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}} = \dfrac{a}{b} \\ ax + by = I \end{cases} \tag{3.3.3}$$

Applying theorem 3.3.2 directly, we have the (minimizing) cost function:

$$C(x) = [96(\frac{\beta}{5\alpha})^{\frac{-\beta}{\alpha+\beta}} + 480(\frac{\beta}{5\alpha})^{\frac{\alpha}{\alpha+\beta}}]x^{\frac{1}{\alpha+\beta}} \tag{3.3.4}$$

Let $\alpha = 0.555$, $\beta = 0.450$; then: the cost function becomes

$$C(x) = 392.56x^{0.995} \tag{3.3.5}$$

where x is the number of parking spaces to construct.

If we build new parking lots to meet all the demand, the goal is to compute the overall profit P (total revenue of the added parking lots - total cost of adding new parking lots) at each district for a duration of $T = 1$ year(s):

- (a) $P \geq 0 \Rightarrow$ should build parking space
- (b) $P < 0 \Rightarrow$ should not build new parking space

However, due to the limiting space on Brown's campus, we cannot build all of the parking lots needed. We also propose to find the minimum amount of parking spot that should be built in each district such that the income from those parking slots over $T = 1$ years will outweigh the cost. Specifically, we find the least x such that

$$R(x) - C(x) > 0$$

where x is the possible number of new parking lots to build, $R(x)$ is the revenue function of x, calculated according to the model in the previous sections 3.1 and 3.2, and $C(x)$ is the cost function of x.

4 Results

To reiterate our assumptions in section 2, we only consider parking problems for permanent parking (spots that are assigned to an individual for the whole season) and visitor parking (spots that are available to visitors and charged by the hours).

4.1 Visitor Parking Spots Allocation

As described in Section 3.2, we want to allocate a specific number of spots in the south part of central campus [4] to maximize profit and the number of spaces occupied. Since the demand in central-south district is always bigger than the supply, no matter what time of year it is, it is clear that all spots should be occupied. Therefore, we only consider the problem of maximizing profit.

But as stated in Section 2, the number of expected visitors on an average day can be very different at different time of year [5]. As a result, the result here must be nuanced and separated by season, as well as weekdays and weekends.

By using a python program [6] to iterate over all possible permanent-visitor parking split, we found the best allocation strategy below.

Time	Visitor Parking Spots Allocated	maximum daily profit
fall weekday	20	\$1403
fall weekend	7	\$1313
spring weekday	273	\$2920
spring weekend	96	\$1732

Table 6: The allocation of visitor parking spots in central-south district

The result shows how many visitor parking spots should be allocated inside the region of the central-south district. It's up to the administration where exactly these spots should be placed in

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[4] we assumed that visitor can only park in this district

[5] the exact number can be found in Section 2

[6] see appendix A for function find_num_visitor_spots()

the district. But wherever they end up, they will lead to the maximum profit as we assumed that visitors will treat parking spots in the same region equally.

It is also important to check whether the allocation strategy in table 6 is reasonable. First of all, the visitor spots reserved in all seasons is smaller than the existing parking slots in the district. This means the allocation strategy is feasible. Second, we need to make sure that most of the visitors at brown who needs to park do end up getting a spot to better their visiting experience.

To do this, we take on the assumptions in Section 2 that the speed at which visitors come to and leave campus follow a normal distribution, with the mean located at 11 am and 1 pm respectively. Based on this assumption, we used a Python program [7] to find the maximum number of visitors on campus at any certain time, and the results are presented below in table 7

	the time when visitor reaches max	maximum number of cars
fall weekday	12 pm	20
fall weekend	12 pm	10
spring weekday	12 pm	273
spring weekend	12 pm	136

Table 7: The time of the day at which number of visitors on campus reach it's peak and the number of visiting cars on campus at that time

Comparing table 6 and table 7, we can see that even at the busiest hour of the day on campus, most of the visitor will be able to get a parking spot. This means that for most of the time of the day, most of the visitors, if not all of them, will be able to get a parking spot and have a good visiting experience at Brown.

4.2 Permanent Parking Spots Allocation

By the assumptions in Section 2, faculty, staff, and emeriti (collectively referred to as employees below) are only willing to park within their own district, whereas undergrads, grad students, and medical students (collectively referred to as students below) are willing to park in the neighboring district if a spot is assigned to them.

Again, since the demand is always greater than the supply in any district, we conclude that all parking spots will be taken and only need to consider maximizing profit. Following the strategy outlined in claim 3.1.1, we first assign spots to employees according to existing demand, and then assign the spots left over to students if there are any.

Since the number of visitors are different for different seasons and days of the week, the number of parking spaces allocated for visitor parking will be different according to results in table 6. As a result, parking spaces allocated for permanent parking will also be different. We need to nuance the result of permanent parking into different seasons, as shown below.

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[7] see appendix A for function max_visitors()

District	Class	Demand	Supply	Allocated Spots	Still Need Spots
Nelson	employee	255	300	255	0
	students	168		45	123
Central-North	employee	255	200	200	55
	students	168		0	168
Central-Mid	employee	255	100	100	155
	students	168		7(central-south)	161
Central-South	employee	255	460	255	0
	students	168		168	0
	visitors	20		20	0
Down-town	employee	514	600	514	0
	students	120		86+17(central-south)	17

Table 8: Fall weekday parking allocation. The 1st column specify the district, the 2nd the class of people, the 3rd the demand in number of parking spots from each class, the 4th the existing number of parking spots, the 5th the number of allocated parking spots to each class (the numbers followed by parenthesis shows and allocation from another district indicated in the parenthesis), and the 6th number of parking spots still needed by each class

4.3 Adding Extra Parking Spots

To start with, we assume that a large number of new parking lots can be built in each district to satisfy all the existing demand. We compute the profit after 1 year for each district. Since the demand is different at different time, we take the maximum among those when computing the cost.

Using the model in Section 3.3, we obtain the results in table 12.

From this, we can see that:

- If we consider the profit of each district separately, then it is profitable to build new parking lots at Nelson, Central-North, and Central-South since the additional income outweigh the cost.
- If we consider the profit of all added parking lots as a whole, then building all of them is worth it since \$589.49 will be the net profit after 1 year.

However, due to the limiting spaces on campus, it's impractical to build all of the parking spaces needed. Assume that the new parking spots are allocated to different classes of people according to the proportion of their demand, our next goal is to find the number of spots x that should be added in each district such that profit $P > 0$ after 1 year.

For instance, in Nelson, the goal is to find the minimum integer $0 < x < 123$ such that:

$$0.25 \times 10 \times 7 \times (14 + 15)x - 392.56x^{0.995} > 0 \tag{4.3.1}$$

District	Class	Demand	Supply	Allocated Spots	Still Need Spots
Nelson	employee	255	300	255	0
	students	168		45	123
Central-North	employee	255	200	200	55
	students	168		0	168
Central-Mid	employee	255	100	100	155
	students	168		22(central-south)	146
Central-South	employee	255	460	255	0
	students	168		168	0
	visitors	10		7	3
Down-town	employee	514	600	514	0
	students	120		86+30(central-south)	4

Table 9: Fall weekend parking allocation. Columns represent the same things as table 8

District	Class	Demand	Supply	Allocated Spots	Still Need Spots
Nelson	employee	255	300	255	0
	students	168		45	123
Central-North	employee	255	200	200	55
	students	168		0	168
Central-Mid	employee	255	100	100	155
	students	168		0	168
Central-South	employee	255	460	187	68
	students	168		0	168
	visitors	273		273	0
Down-town	employee	514	600	514	0
	students	120		86	34

Table 10: Spring weekday parking allocation. Columns represent the same things as table 8

District	Class	Demand	Supply	Allocated Spots	Still Need Spots
Nelson	employee	255	300	255	0
	students	168		45	123
Central-North	employee	255	200	200	55
	students	168		0	168
Central-Mid	employee	255	100	100	155
	students	168		0	168
Central-South	employee	255	460	255	0
	students	168		109	54
	visitors	136		96	40
Down-town	employee	514	600	514	0
	students	120		86	34

Table 11: Spring weekend parking allocation. Columns represent the same things as table 8

District	number of added Spots	Revenue after 1 year ($)	Cost ($)	Profit ($)
Nelson	123	48011.82	47136.97	874.85
Central-North	223	93824.57	85205.84	8618.73
Central-Mid	323	143056.89	123186.24	19870.55
Central-South	236	65118.43	90147.46	-25029
Downtown	34	9368.16	13113.77	-3745.61
Total	939	359379.87	358790.38	589.49

Table 12: Profits at each district after 1 year. The 1st column specify the district, the 2nd the quantity were built, the 3rd the revenue after 1 year, the 4th the cost of meeting the need, the 5th the overall profit (added income - cost) after 1 year

If a solution x_0 exists, then the actual number of parking spaces to be added should be between x_0 and 123, depending on other limiting factors such as available space on campus. Otherwise new parking spaces should not be added.

Similarly, we obtained the results in table 13. However, we do note that it is purely a coincidence that as long as new parking lots are built in the upper 3 districts, positive profit is earned. If the sum of coefficients of the Cobb-Douglas production $(\alpha + \beta)$ is farther away from 1, then the lower bounds for this three will not be identically 1. For example, if we take $\alpha + \beta = 1.2$, at least 6 parking slots would need to be built to make positive profit.

Therefore, as long as new parking lots are built in Nelson, Central-North, and Central-Mid, Brown will make a positive profit. Hence the strategy is to build many new parking spaces in the above 3 districts as space allows, as more parking lots creates more profit.

For the two other districts, Central-South and Downtown, we do not recommend building new parking lots since the cost would always outweigh the additional income.

District	smallest number of added spots to make positive profit
Nelson	1
Central-North	1
Central-Mid	1
Central-South	Solution does not exist
Downtown	Solution does not exist

Table 13: Lower bound of number of parking lots added that can make positive profit. The 1st column specify the district, the 2nd the lower bound

5 Discussion

In this section, we summarize our solution for the Brown parking problem, and point out the strength and weakness of our model as well as possible improvements.

5.1 conclusion

To summarize, our solution to the Brown University parking problem can be divided into three parts. Firstly, for permanent individual parking permits, we have shown that they should be issued to faculty, emeriti, and staff according to their demand, and then issued to students if there are still spaces left. Secondly, for allocation of parking spots for visitor parking, we found that Brown should allocate different number of visitor parking spots based on different time of the year and time of the week. We have calculated the exact number of the optimal allocation for spring and fall, weekday and weekends. We have proved that this allocation strategy for visitor and permanent parking will bring a maximum number of occupied spots and maximum profit. Finally, we have compared the cost of building new parking spots against the added income, and concluded that new parking spots should be built north of Waterman Street. The more parking slots built, the greater the profit.

5.2 Strength and Weakness

5.2.1 Strength

One big advantage of our model is that it is easy to generalize. Although the model is developed based on situations of Brown University, many other schools may confront similar problems. One advantage of our model is that it can be applied to other schools by simply modifying some parameters such as hourly rates, numbers of people, and people-to-car ratios.

Moreover, our model is effective in capturing the different parking needs among different classes. We have considered the distribution of different classes of people across campus, and assigned them the best parking spots as possible. This maximizes the utility of every single individual, while also maximizing the parking income profit of Brown, creating a win-win situation.

Finally, our model takes into account the different demand in different seasons and days of the week. We adjust our allocation strategy at different time to maximize both the parking spots occupied and the parking income.

5.2.2 Weakness

Even though the current model has many advantages, we do acknowledge its weaknesses.

To begin with, we made many strict assumptions to simply the model. For instance, in section 3.3, production is assumed to be continuously differentiable to reduce the mathematical complexity. However, most functions are not smooth and continuously differentiable in the real world. Besides that, We also simplified the data. For example, the real hourly rates at Brown is different in terms of parking time and car owner's salary, but we simplified them in the models to be uniform.

Secondly, some choices of model parameters are not rigorous. For example, most of the people-to-car ratios for each class of people are based purely on the author's experience, with no reliable sources. Even though the relative ranking of those ratios are possibly correct, the exact value may not, and the numerical value has more effects on the accuracy of model than their relative rank.

In addition, when estimating the lower bound of number of new parking spots that needs to be build, the relationship between the demand of parking spaces among different classes of people is assumed to be linear. This relationship is too simple to produce an accurate description.

5.3 Direction for Future Work

We recognize that although we have presented good results with the current model, there are many improvements that can be made.

One direction for future work could be to include street parking and the parking in the surrounding parts of Providence in our model. The demand for street parking is particularly difficult to estimate, and the drivers that uses street parking is a lot more diverse than the 7 classes of people we consider. This needs to be handled with care.

Another improvement is modeling the distribution of the 7 classes of people across campus more accurately, in replacement of the uniform distribution we assumed in this model.

Thirdly, with the adding parking lots model narrowing down the range of newly added parking lots, the bounds can be further narrowed if data of spacing available for building parking lots were found (i.e. limiting factors other than financial ones). Also, the production function may not be as simple as a Cobb-Douglas function; replacement of it may improve the accuracy of the model.

In addition, we limited visitor parking area to only one of the five districts. Better results might be found if we lift this restriction.

A final direction is to model the distribution of parking spaces with more precision in each of the district, allowing for more nuanced calculation of the supply and demand of parking spots.

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A Python Code

```
from scipy.stats import norm


T_START = 8  # start time of when ppl arrive
T_END = 6 + 12  # end time of when ppl leave
IN_RUSH = 11  # the time when most ppl come to Brown
OUT_RUSH = 1 + 12  # the time when most ppl leave Brown


def num_visitors(t, total_visitors):
    """
    get the number of visitors currently on campus at a certain time t
    v_in ~ total_tourist * N(IN_RUSH, 1)
    v_out ~ total_tourist * N(OUT_RUSH, 1)
    :param t: int, the time of day in 24-hour time
    :param total_visitors: the total number of expected visitors on that whole day
    :return: the number of visitors at time t
    """
    return (norm.cdf(t, loc=IN_RUSH, scale=1) - \
        norm.cdf(t, loc=OUT_RUSH, scale=1)) * total_visitors


def max_visitors(total_tourist):
    """
    find the maximum amount of visitor at brown at a certain time in day
    :param total_tourist: int, the total number of tourists on one day
    :return: the time achieving the max, the max amount
    """
    running_max = 0
    time_at_max = None
    for t in range(T_START, T_END + 1):
        step_max = num_visitors(t=t, total_visitors=total_tourist)
        if step_max > running_max:
            running_max = step_max
            time_at_max = t
    return time_at_max, running_max
```

```
# in dollar, faculty pay per day to park
FAC_HOURLY_RATE = 17.71 / 7 / (T_END - T_START)

VIS_WEDAY_HOURLY_RATE = 3  # visitor weekday
VIS_WEEND_HOURLY_RATE = 2  # visitor weekend


def find_num_visitor_spots(total_spots,
                           total_visitor,
                           vis_rate,
                           faculty_rate=FAC_HOURLY_RATE):
    """
    find the optimal number of parking spots to allocate
    specifically to visitors to achieve max total profit
    this is a brute force solution: iterate over
    all possibilities and find the max
    :param total_spots: int, the number of total
                        parking spots in a particular district
    :param total_visitor: int, the number of total visitors
                          expected in the whole day
    :param vis_rate: in dollars, visitor pay per hour to park
    :param faculty_rate: in dollars, faculty pay per hour to park
    :return: maximun daily profit, number of allocated
            visitor parking spots at max profit
    """
    max_profit = 0
    optimum_vis_spots = 0
    for visitor_spots in range(0, total_spots, 1):
        daily_profit = 0
        for t in range(T_START, T_END + 1):
            num_vis = num_visitors(t=t, total_visitors=total_visitor)
            daily_profit += min(num_vis, visitor_spots) * vis_rate +
                          (total_spots - visitor_spots) * faculty_rate
        if daily_profit > max_profit:
            max_profit = daily_profit
            optimum_vis_spots = visitor_spots
    return max_profit, optimum_vis_spots
```

B Parking Map

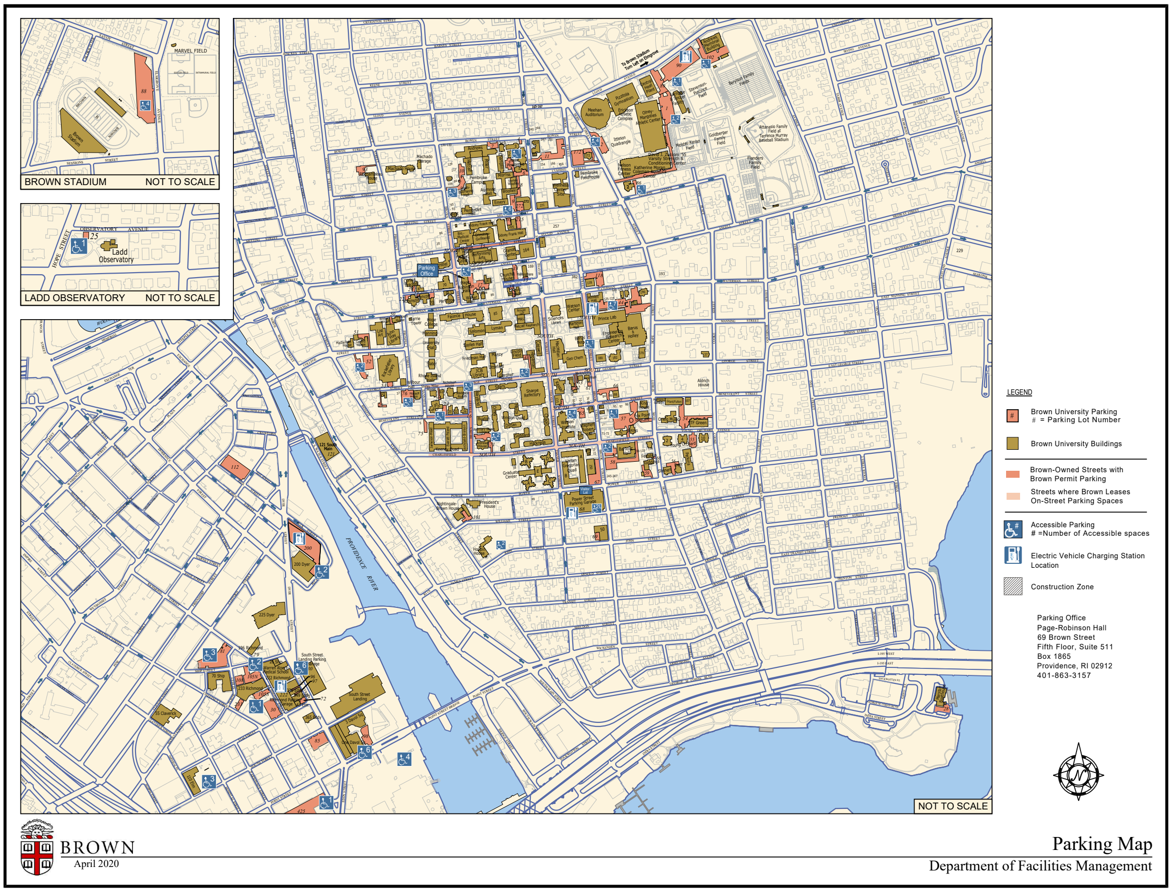

Figure 1: Parking Map of Brown University as provided by the Brown Transportation Office [2]